Approximate calculation of corrections at NLO and NNLO. *

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For processes involving structure functions and/or fragmentation functions, arguments that there is a part that dominates the NLO corrections are briefly reviewed. The arguments are tested against more recent NLO and in particular NNLO calculations.

1. THE DOMINANT PART AND ITS IMPLICATIONS

For many unpolarized and polarized inclusive reactions calculations have now been carried in next-to-leading order (NLO) in the running coupling $\alpha_s(Q)$ and in a few of them in next-to-next-to-leading order (NNLO). All analytic NNLO results are very complicated and the same holds for the NLO ones when the leading order (LO, Born) subprocess corresponds to a 4-point function.

For processes involving structure functions and/or fragmentation functions, in [1] it was argued that there is a part that dominates the NLO; and this was used to explain the fact that, in a number of the then existing NLO calculations, plotted against the proper kinematic variable, in a wide range, the cross section was almost a constant multiple of the Born.

Here we extend these considerations to a number of NLO calculations carried in the meantime, in particular on polarized reactions, as well as to the existing NNLO ones.

To briefly review the essential ideas of [1], consider the NLO contribution of the subprosses $a(p_1) + b(p_2) \to \gamma(q) + d$ to the large-p_T process $A + B \to \gamma + X$:

$$E\frac{d\sigma}{d^3p} = \frac{\alpha_s(\mu)}{\pi}\int dx_a dx_b F_{a/A}\left(x_a, M\right) F_{b/B}\left(x_b, M\right)\left[\hat{\sigma}_B\delta\left(1+\frac{t+u}{s}\right) + \frac{\alpha_s(\mu)}{\pi} f\theta\left(1+\frac{t+u}{s}\right) + cross.\,term\right] \quad (1)$$

where $F_{a/A}$, $F_{b/B}$ are parton momentum distributions, μ and M are of $O(p_T)$,

$$s = \left(p_1+p_2\right)^2, \quad t = \left(q-p_1\right)^2, \quad u = \left(q-p_2\right)^2$$

and σ_B and f are functions of s, t, u corresponding to the Born and the higher order correction (HOC). Introducing the dimensionless variables

$$v = 1 + t/s, \quad w = -u/\left(s+t\right) \quad (2)$$

($s{+}t{+}u = sv(1{-}w)$), the HOC have the following overall structure:

$$f\left(v, w\right) = f_s\left(v, w\right) + f_h\left(v, w\right)$$

where

$$f_s\left(v, w\right) = a_1(v)\delta(1-w) + b_1(v)\frac{1}{(1-w)_+} + c(v)\left(\frac{ln(1-w)}{1-w}\right)_+ + \left(a_2(v)\delta(1-w) + b_2(v)\frac{1}{(1-w)_+}\right) ln\frac{s}{M^2} \quad (3)$$

$f_h\left(v, w\right)$ contains no distributions and, in general, is very complicated.

Now denote by σ_s and σ_h the contributions of f_s and f_h to $Ed\sigma/d^3p$ and consider the ratio

$$L = \sigma_h/(\sigma_s + \sigma_h); \quad (4)$$

then, for fixed total c.m. energy $\sqrt{S}$, as p_T (or $x_T \equiv 2p_T/\sqrt{S}$) increases, L decreases.

To see the reason, consider a plot of x_b vs x_a (Fig. 1 of [1a]). The integration region in (1) is

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bounded by $w=1$, $x_a=1$ and $x_b=1$. Now, for x not too small, $F_{a/A}(x,M)$ behaves like $(1-x)^n$; with A =proton, n is fairly large (≥ 3); also due to scale violations, n increases as p_T increases. Then contributions arising from the region away from $w=1$ are supressed by powers of $1-x_a$ and/or $1-x_b$. Now, in f_s, the terms $\sim \delta(1-w)$ contribute at $w=1$ (and so does $\hat{\sigma}_B$) whereas the rest give a contribution increasing as $w \to 1$. On the other hand, the multitude of terms of f_h contribute more or less uniformly in the integration region $\theta(1-w)$ and their contribution σ_h is suppressed. As x_T increases at fixed S, the integration region shrinks towards $x_a = x_b = 1$ (Fig. 1 of [1a]) and the suppression of σ_h increases.

The mechanism is tested by writing the distributions in the form [1a]:

$$F_{a/A}(x,M) = F_{b/B}(x,M) = (1-x)^N \tag{5}$$

and choosing a fictitious $N >> n$ or $0 < N << n$. Then the ratio L in the first case decreases faster, in the second slower.

Neglecting $f_h(v,w)$ and with the rough approximations $1/(1-w)_+ \sim \delta(1-w)$, $(ln(1-w)/(1-w))_+ \sim \delta(1-w)$ we obtain in (1): $f \approx \delta(1-w)$ resulting in $Ed\sigma/d^3p$ of roughly the same shape as $Ed\sigma_{Born}/d^3p$.

At NLO the Bremsstrahlung (Brems) contributions to f_s are determined via simple formulae [1]: E.g. for $gq \to \gamma q$ the Brems contributions arise from products of two graphs $gq \to \gamma qg$. If in both graphs the emitted g arises from initial partons (g or q), the contribution in $n = 4-2\varepsilon$ dimensions is

$$\frac{d\sigma_{init}}{dvdw} \sim T_0^{(gq)}(v,\varepsilon) N_c \left(-\frac{2}{\varepsilon}\right)\left(\frac{v}{1-v}\right)^{-\varepsilon} (1-w)^{-1-2\varepsilon}\left(1+\varepsilon^2\frac{\pi^2}{6}\right) \tag{6}$$

where $T_0^{(gq)}(v,\varepsilon)$ is essentially the Born cross section in n dimensions. If in at least on of the graphs the emitted g arises from the final parton (q), then

$$\frac{d\sigma_{fin}}{dvdw} \sim T_0^{(gq)}(v,\varepsilon) C_F v^{-\varepsilon}(1-w)^{-1-\varepsilon}\tilde{P}_{qq}(\varepsilon) \tag{7}$$

where

$$\tilde{P}_{qq}(\varepsilon) = \frac{\Gamma(1-2\varepsilon)}{\Gamma^2(1-\varepsilon)}\int_0^1 y^{-\varepsilon}(1-y)^{-\varepsilon} P_{qq}(y,\varepsilon)$$

and $P_{qq}(y,\varepsilon) = 2/(1-y)-1-y-\varepsilon(1-y)$, the split function in n dimensions ($y<1$). Expanding

$$(1-w)^{-1-\varepsilon} = -\frac{1}{\varepsilon}\delta(1-w) + \frac{1}{(1-w)_+} - \varepsilon\left(\frac{ln(1-w)}{1-w}\right)_+ + O(\varepsilon^2)$$

as well as $(v/(1-v))^{-\varepsilon}$ and $v^{-\varepsilon}$ in powers of ε we determine the contributions. The singular terms $\sim 1/\varepsilon^2$ and $1/\varepsilon$ cancel by adding the loop contributions and proper counterterms.

2. FURTHER NLO CALCULATIONS

In addition to the examples presented in Refs [1], the following are some NLO studies supporting the ideas of Sect. 1:

(a) Heavy quark Q production in $p\bar{p}$ collisions [2]. The cross sections $d\sigma/dydp_T^2$ versus p_T of Q for several rapidities y and for $m_Q = 5$, 40 and 80 GeV at $\sqrt{S} = 0.63$ and 1.8 TeV are a constant multiple of the LO one (Figs 7-12). See also [3] Fig. 10.15. In all the cases the verification is striking.

(b) Large p_T W and Z production in $p\bar{p}$ collisions [4]. At $\sqrt{S} = 0.63$ and 1.8 TeV, for $p_T \geq 80$ GeV the cross sections $d\sigma/dp_T^2$ are also almost a constant multiple of the LO (Figs 7 and 8).

Regarding NLO results for polarized reactions we mention the following:

(a) Polarized deep inelastic Compton scattering [5], in particular the contribution of the subprocess $\vec{\gamma}\vec{p} \to \gamma q$ to large p_T $\vec{\gamma}\vec{q} \to \gamma + X$. At $\sqrt{S} = 27$ and 170 GeV, for $x_T \geq 0.15$, it is $L < 0.28$ and for sufficiently large x_T, L decreases as $x_T \to 1$ (Ref. 5, Fig. 4). Also, denoting by $\sigma^{(k)}$ the $O(\alpha_s^k)$, $k=0,1$, contributions of $\vec{\gamma}\vec{q} \to \gamma q$ to $Ed\sigma/d^3p$, for $0.2 \leq x_T \leq 0.8$ the factor $K_{\gamma q} = (\sigma^{(0)}+\sigma^{(1)})/\sigma^{(0)}$ is found to differ little from a constant.

(b) Large p_T direct γ production in longitudinally polarized hadron collisions [6, 7]. Here of interest are the $O(\alpha_s^k)$, $k = 1, 2$, of the subprocess $\vec{g}\vec{q} \to \gamma q$. As x_T increases, the ratio $-\sigma_h/\sigma_s$ steadily decreases (Ref. 6, Fig. 10). The factor $K_{gq} = (\sigma^{(1)} + \sigma^{(2)})/\sigma^{(1)}$ is not constant, but increases moderately (Fig. 2).

(c) Lepton pair production by transversely polarized hadrons [8, 9]. At fixed S, with increasing $\sqrt{\tau} = M_{l^-l^+}/\sqrt{S}$, the ratio σ_h/σ_s is found again to decrease (Ref. 8, Fig. 3). Again, the K-factor is not constant, but increases moderately (Ref. 8, Fig. 1).

The considerations of Sect. 1 explain also the following fact: Taking as example large p_T $\vec{p}\vec{p} \to \gamma + X$, at NLO, apart from the HOC of the dominant subprocess $\vec{g}\vec{q} \to \gamma q$, there are contributions from the extra subprocesses $\vec{g}\vec{g} \to q\bar{q}\gamma$ and $\vec{q}\vec{q} \to qq\gamma$. In general, these are found to be small (Ref. 6, Figs 3, 4 and 5). The reason is that the extra subprocesses possess no terms involving distributions (no loops and vanishing contributions of the type (6) and (7)).

3. NNLO CALCULATIONS

NNLO calculations have been carried for Drell-Yan (DY) production of lepton pairs, $W^\pm$ and Z, and for the deep inelastic (DIS) structure functions $F_j(x, Q^2)$, $j = 1, 2$ and L. Now the parts involving distributions contain also terms of the type $(ln^i(1-w)/(1-w))_+$, with $i = 2$ and 3 and w a proper dimensionless variable. Calculations are carried using the set $S - \overline{MS}$ of [10].

Beginning with DY, we are interested in the process $h_1 h_2 \to \gamma^* + X \to l^+ l^- + X$ and to the cross section

$$d\sigma(\tau, S)/dQ^2 \equiv \sigma(\tau, S) \qquad (8)$$

where $\tau = Q^2/S$ with $\sqrt{S}$ the total c.m. energy of the initial hadrons h_1, h_2 and $\sqrt{Q^2}$ the γ^* mass [11, 12]. Here we deal with the subprocess $q + \bar{q} \to \gamma^*$ and its NLO and NNLO corrections [11]. For DY, $w \sim \tau$.

Denote by $\sigma^{(k)}(\tau, S)$, $k = 0, 1, 2$, the $O(\alpha_s^k)$ part of $\sigma(\tau, S)$, by $\sigma_s^{(k)}$ the part of $\sigma^{(k)}$ arising from distributions and by $\sigma_h^{(k)}$ the rest. Defining

$$L^{(k)}(\tau, S) = \sigma_h^{(k)}(\tau, S)/\sigma^{(k)}(\tau, S) \qquad (9)$$

Fig. 1 shows $L^{(k)}$, $k = 1, 2$, as functions of τ for $\sqrt{S} = 20\ GeV$. Clearly, for $\tau > 0.3$: $L^{(1)} \le 0.16$ and $L^{(2)} \le 0.4$.

It is of interest also to see the percentage of $\sigma_h^{(k)}$ of the total cross section determined up to $O(\alpha_s^k)$. Fig. 1 also shows the ratios $\sigma_h^{(1)}/(\sigma^{(0)} + \sigma^{(1)})$ and $\sigma_h^{(2)}/(\sigma^{(0)} + \sigma^{(1)} + \sigma^{(2)})$ for the same $\sqrt{S}$; clearly, for $\tau \ge 0.2$ both ratios are less than 0.1.

Now we turn to DIS [13, 14] and present results for the contribution to the structure function $F_2(x, Q^2)$ of the d-valence quark distribution. We will deal with the subprocess $q + \gamma^* \to q$ and its NLO and NNLO corrections [14]. For DIS, $w \sim x$.

Denote by $F^{(k)}(x, Q^2)$, $k = 0, 1, 2$, the $O(\alpha_s^k)$ contribution, by $F_s^{(k)}$ the part of $F^{(k)}$ arising from distributions and by $F_h^{(k)}$ the rest. Defining

$$L^{(k)}(x, Q^2) = F_h^{(k)}(x, Q^2)/F^{(k)}(x, Q^2) \qquad (10)$$

Fig. 2 presents $L^{(k)}(x, Q^2)$, $k = 1, 2$, as functions of x for $\sqrt{Q^2} = 5\ GeV$. Now, for $x \le 0.5$ $L^{(1)}$ is not small, but this is due to the fact that $F_s^{(1)}$ changes sign and $F_h^{(1)}$ stays > 0, so at $x \approx 0.3$ $F^{(1)}$ vanishes. On the other hand, at $x \ge 0.3$, $L^{(2)}$ is less than 0.2.

Fig. 2 also shows the ratios $F_h^{(1)}/(F^{(0)} + F^{(1)})$ and $F_h^{(2)}/(F^{(0)} + F^{(1)} + F^{(2)})$ for the same Q^2; for $x \ge 0.3$ both ratios are less than 0.08.

The effect of neglecting $\sigma_h^{(k)}$ in DY or $F_h^{(k)}$ in DIS is shown in Fig. 3. In DY, denoting

$$K_s = (\sigma^{(0)} + \sigma_s^{(1)} + \sigma_s^{(2)})/\sigma^{(0)}$$
$$K = (\sigma^{(0)} + \sigma^{(1)} + \sigma^{(2)})/\sigma^{(0)} \qquad (11)$$

we show $K_s(K)$ by solid (dashed) line at $\sqrt{S} = 20$ GeV (upper part). Clearly, as $\tau \to 1$, $K_s \to K$, and for $\tau > 0.3$ the error is less than 12%. In DIS, denoting by K_s and K the K-factors of (11) with $\sigma^{(k)}$ replaced by $F^{(k)}$, we show K_s and K at $\sqrt{Q^2} = 5\ GeV$ (lower part). Again, as $x \to 1$, $K_s \to K$. Now, in spite of the fact that $L^{(k)}$ is, in general, not small, K_s differs from K even less. The reason is that the NLO and NNLO corrections are smaller than in DY, and so are $F_s^{(k)}/F^{(0)}$.

4. CONCLUSIONS

The above discussion and examples show that for processes involving structure and/or fragmentation functions, for not too small values of a proper kinematic variable (x_T for large-p_T reactions, τ for DY, x for DIS), with reasonable accuracy one can retain only the part of the differential cross section arising from distributions (dominant part).

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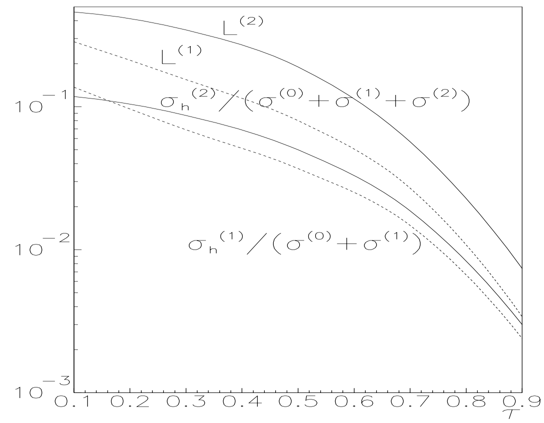


Fig. 1

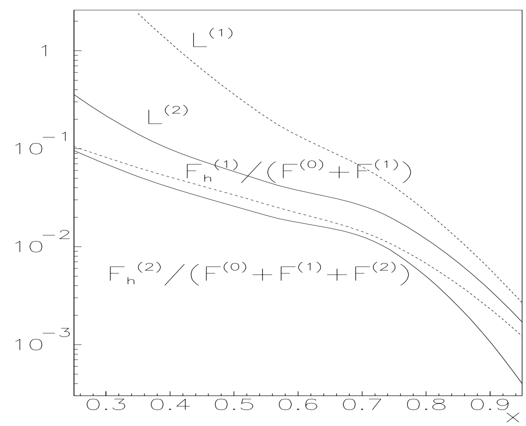


Fig. 2

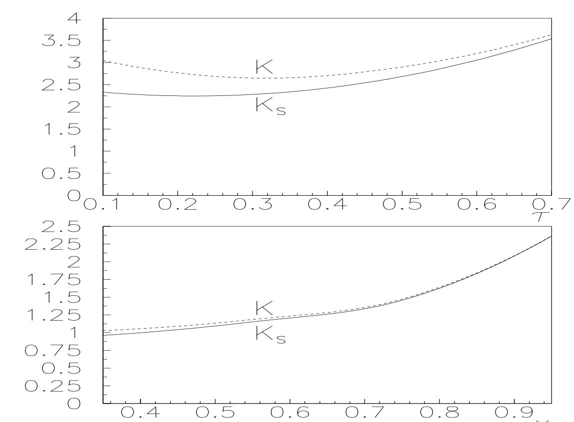
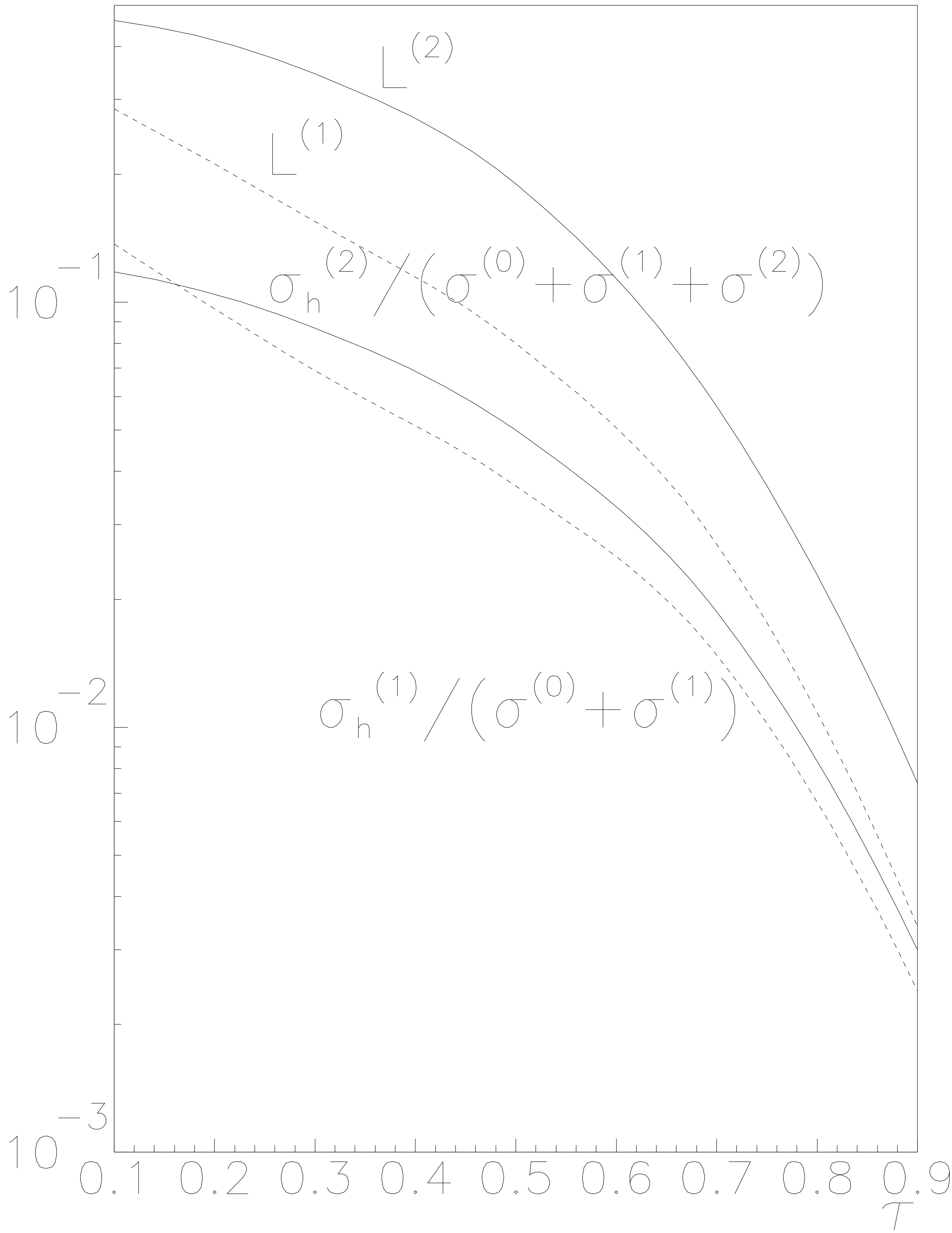
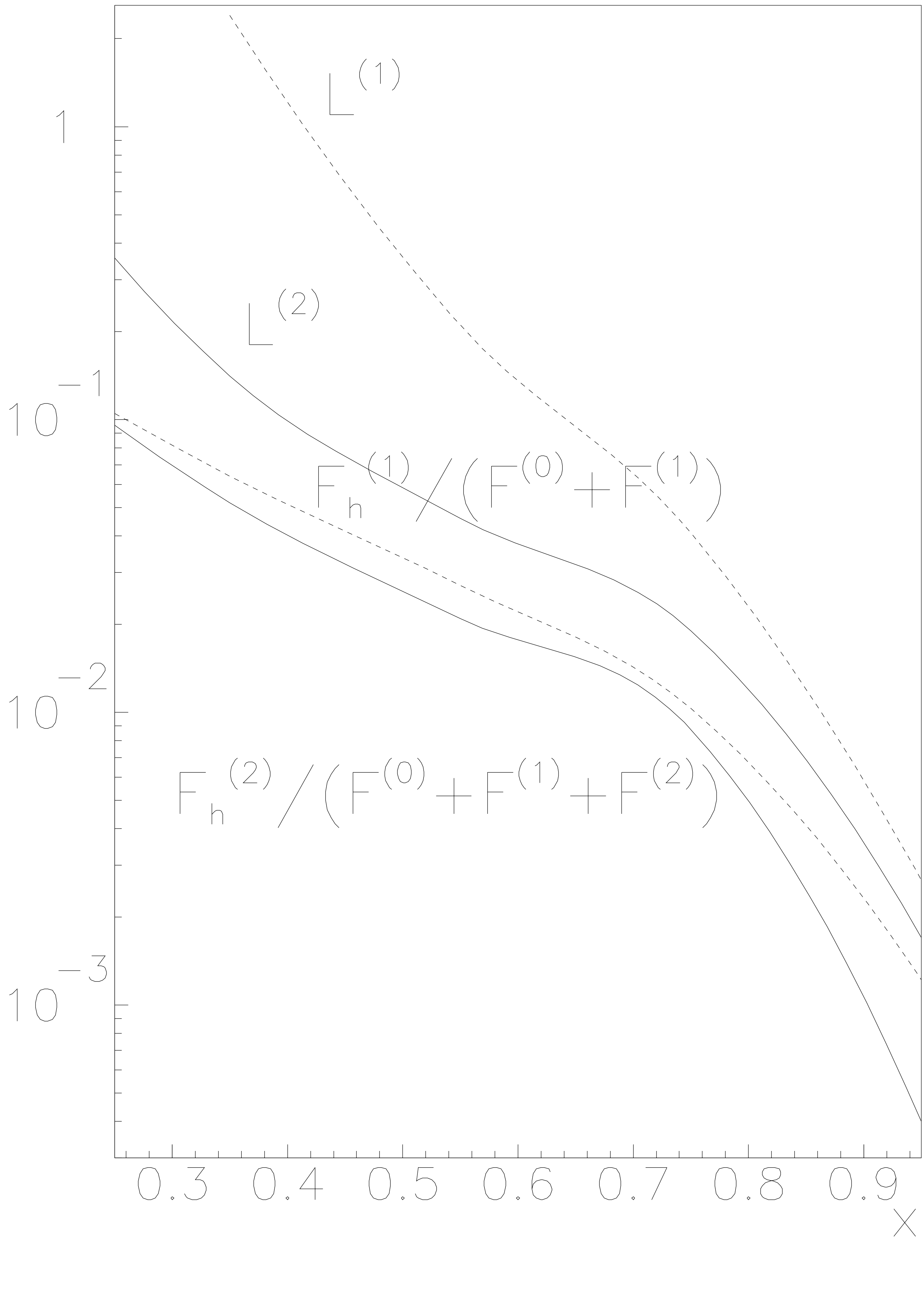


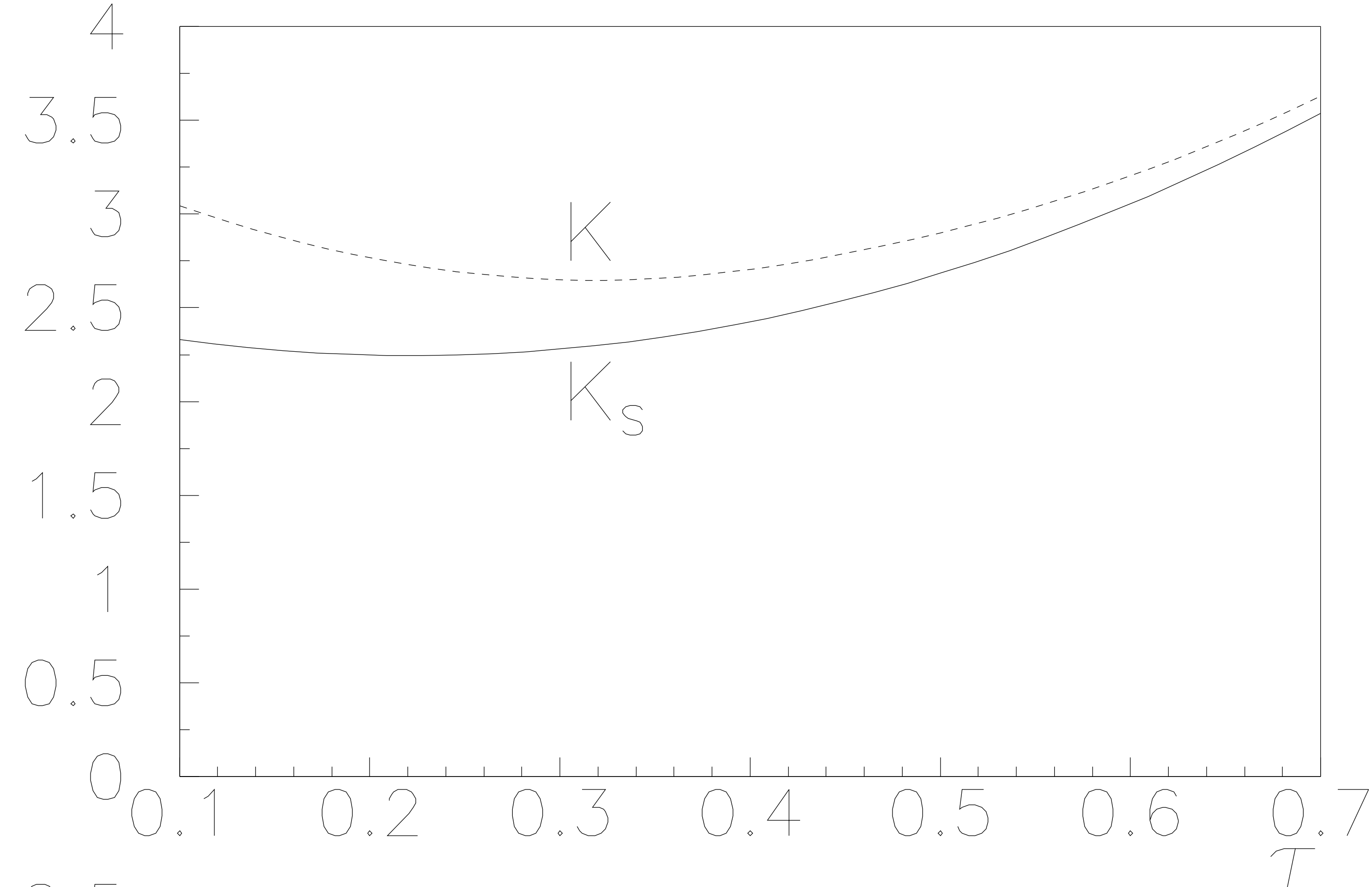
Fig. 3



$L^{(2)}$
$L^{(1)}$
$\sigma_h^{(2)}/(\sigma^{(0)}+\sigma^{(1)}+\sigma^{(2)})$
$\sigma_h^{(1)}/(\sigma^{(0)}+\sigma^{(1)})$
10^{-1}
10^{-2}
10^{-3}
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
τ



$L^{(1)}$
$L^{(2)}$
$F_h^{(1)}/(F^{(0)}+F^{(1)})$
$F_h^{(2)}/(F^{(0)}+F^{(1)}+F^{(2)})$
1
10^{-1}
10^{-2}
10^{-3}
0.3
0.4
0.5
0.6
0.7
0.8
0.9
X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3
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2
1.5
1
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K_S
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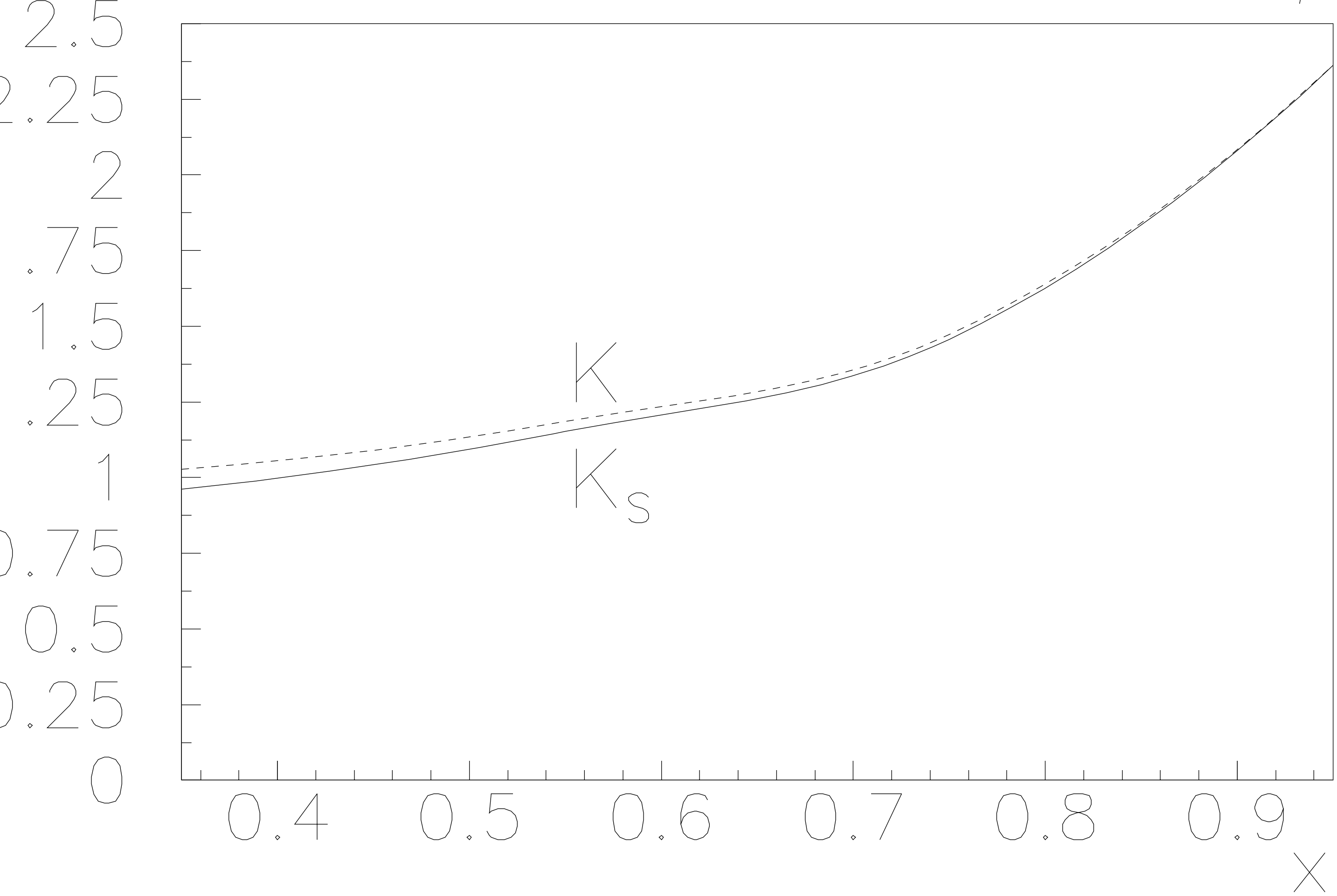
0.5
0.6
0.7
T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1.5
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1
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K_S
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0.6
0.7
0.8
0.9
X